AVIRAL
JEE | NEET | FOUNDATIONS

WEEKLY TEST TYJ TEST - 30 B
SOLUTION Date 08-12-2019

[PHYSICS]

1. Let displacement equation of particle executing SHM is

$$y = a \sin \omega t$$

As particle travels half of the amplitude from the equilibrium position, so

$$y = \frac{a}{2}$$

Therefore, $\frac{a}{2} = a \sin \omega t$

or $\sin \omega t = \frac{1}{2} = \sin \frac{\pi}{6}$

or $\omega t = \frac{\pi}{6}$

or $t = \frac{\pi}{6\omega}$ or

or $t = \frac{\pi}{6\left(\frac{2\pi}{T}\right)}$ $\left(\text{as } \omega = \frac{2\pi}{T}\right)$ 60. Given,

and

or $t = \frac{T}{12}$

Hence, the particle travels half of the amplitude from equilibrium in $\frac{T}{12}$ s.

2.
3.

4. Time period of a simple pendulum

$$T = 2\pi \sqrt{\frac{l}{g}}$$

It is independant of the mass of the bob. Therefore time period of the pendulum will remain T.

5.

6. **(b)** Use the law of conservation of energy. Let x be the extension in the spring.

Applying conservation of energy

$$mgx - \frac{1}{2}kx^2 = 0 - 0 \Rightarrow x = \frac{2mg}{k}$$

7. **(c)** The two displacement equations are $y_1 = a\sin(\omega t)$

and $y_2 = b\cos(\omega t) = b\sin\left(\omega t + \frac{\pi}{2}\right)$

$$y_{eq} = y_1 + y_2$$
$$= a\sin\omega t + b\cos\omega t$$
$$= a\sin\omega t + b\sin\left(\omega t + \frac{\pi}{2}\right)$$

Since the frequencies for both SHMs are same, resultant motion will be SHM.

Now $A_{eq} = \sqrt{a^2 + b^2 + 2ab\cos\frac{\pi}{2}}$

8. **(d)** As springs are connected in series, effective force constant

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \Rightarrow k = \frac{k_1 k_2}{k_1 + k_2}$$

Hence, frequency of oscillation is

$$n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{(k_1 + k_2)m}}$$

9. **(c)** $n = \frac{1}{2\pi}\sqrt{\frac{K_{\text{effective}}}{m}}$

Springs are connected in parallel

$$K_{\text{eff}} = K_1 + K_2 = K + 2K = 3K$$

$$\Rightarrow \quad n = \frac{1}{2\pi}\sqrt{\frac{(K+2K)}{m}} = \frac{1}{2\pi}\sqrt{\frac{3K}{m}}$$

10. **(a)** As springs are connected in series, effective force constant

$$\frac{1}{k_{\text{eff}}} = \frac{1}{k} + \frac{1}{k} = \frac{2}{k} \Rightarrow k_{\text{eff}} = \frac{k}{2}$$

Hence, frequency of oscillation is

$$n = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{m}} = \frac{1}{2\pi}\sqrt{\frac{k}{2M}}$$

11. The motion of planets around the sun is periodic but not simple harmonic motion.

12. For freely falling case the effective g is zero, so that frequency of oscillation will be zero.

As $$f = \frac{1}{2\pi}\sqrt{\frac{g_{\text{eff}}}{\lambda}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{0}{\lambda}}$$

$$f = 0$$

13. $x(t) = A\cos(\omega t + \phi)$

where, ϕ is the phase constant.

14. $$y_1 = 5[\sin 2\pi t + \sqrt{3}\cos 2\pi t]$$

$$= 10\left[\frac{1}{2}\sin 2\pi t + \frac{\sqrt{3}}{2}\cos 2\pi t\right]$$

$$= 10\left[\cos\frac{\pi}{3}\sin 2\pi t + \sin\frac{\pi}{3}\cos 2\pi t\right]$$

$$= 10\left[\sin\left(2\pi t + \frac{\pi}{2}\right)\right] \Rightarrow \quad A_1 = 10$$

Similarly, $$y_2 = 5\sin\left(2\pi t + \frac{\pi}{4}\right)$$

$\Rightarrow$ $$A_2 = 5$$

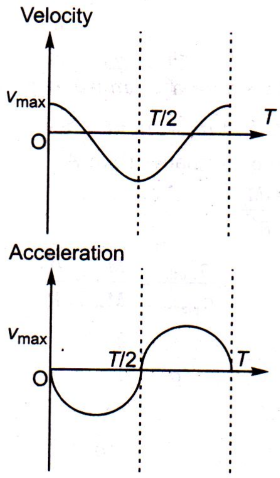
Hence, $$\frac{A_1}{A_2} = \frac{10}{5} = \frac{2}{1}$$

15. Phase difference $\Delta\phi = \phi_1 - \phi_2$

$$= \frac{3\pi}{6} - \frac{\pi}{6}$$

$$= \frac{2\pi}{6} = \frac{\pi}{3}$$

16. In SHM, the acceleration is ahead of velocity by a phase angle $\frac{\pi}{2}$.



$$v = A\omega\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$a = A\omega^2\sin(\omega t + \pi)$$

17. The average acceleration of a particle performing SHM over one complete oscillation is zero.

18. By using $k \propto \frac{1}{l}$

Since, one-fourth length is cut away so remaining length is $\frac{3}{4}$ th, hence k becomes $\frac{4}{3}$ times *ie*, $k' = \frac{4}{3}k$.

19. For the given figure,

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{m}} \quad \ldots(i)$$

$$= \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$$

If one spring is removed, then $k_{eq} = k$ and

$$f' = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{f}{f'} = \sqrt{2}$$

$$f' = \frac{1}{\sqrt{2}} f$$

20. In a complete cycle of SHM, potential energy varies for half the cycle and kinetic energy varies for the other half of the cycle. Thus, for a time period T, the potential energy varies for $\frac{T}{2}$ time.

[CHEMISTRY]

21. Higher the oxidation number of central aotm in oxoacid, higher is the acidic character.

22. HIO_4 H_3IO_5 H_5IO_6
$1 + x - 8 = 0$ $3 + x - 10 = 0$ $5 + x - 12 = 0$
$x = +7$ $x = +7$ $x = +7$

23. Oxidation number of Br_2 (element form) = 0
In BrO_3^-, O.N. of Br + 3(–2) = –1
⇒ O.N. of Br = +5

24. $H_4P_2O_5$: $4 + 2x - 10 = 0$
⇒ x = + 3
$H_4P_2O_6$: $4 + 2x - 12 = 0$
⇒ x = + 4
HP_2O_7: $4 + 2x - 14 = 0$
⇒ x = + 5

26. Lower the reduction potential, better the reducing power. Hence, the correct choice is 'd'

27. $\overset{(0)}{3Cl_2} + 6OH^- \rightarrow \overset{(-1)}{5Cl^-} + \overset{(+5)}{ClO_3^-} + 3H_2O$

28. Higher the reduction potential easier to gain electrons.
Lower the reduction potential easier the loss of electrons.

30. Valence is combining capacity. Hence, valence of sulphur is '2'. Oxidation number is apparent charge which is zero for form O.N. of sulphur element is zero.

32. In $C_2O_4^{2-}$, C-atom has O.N. + 3. In CO_2, C-atom has O.N. +4. Hence, $C_2O_4^{2-}$ is reducing agent

37. Higher the reduction potential, easier is the gain of electrons.

[MATHEMATICS]

41. (c) $\lim_{n\to\infty} \frac{n(2n+1)^2}{(n+2)(n^2+3n-1)} = \lim_{n\to\infty} \frac{4n^3+4n^2+n}{n^3+5n^2+5n-2}$

$$= \lim_{n\to\infty} \frac{n^3\left(4+\frac{4}{n}+\frac{1}{n^2}\right)}{n^3\left(1+\frac{5}{n}+\frac{5}{n^2}-\frac{2}{n^3}\right)} = 4$$

42. (b) $\lim_{n\to\infty} \frac{1}{1+\sqrt{1+\frac{1}{n}}} = \frac{1}{2}$.

43. (b) $\lim_{x\to a} \frac{\sqrt{3x-a}-\sqrt{x+a}}{x-a}$

$$= \lim_{x\to a} \frac{\sqrt{3x-a}-\sqrt{x+a}}{(x-a)} \times \frac{\sqrt{3x-a}+\sqrt{x+a}}{\sqrt{3x-a}+\sqrt{x+a}}$$

$$= \frac{2}{2\sqrt{2a}} = \frac{1}{\sqrt{2a}}$$

Aliter : Apply L-Hospital's rule

$$\lim_{x\to a} \frac{\sqrt{3x-a}-\sqrt{x+a}}{x-a} = \lim_{x\to a} \frac{3}{2\sqrt{3x-a}} - \frac{1}{2\sqrt{x+a}}$$

$$= \frac{3}{2\sqrt{2a}} - \frac{1}{2\sqrt{2a}} = \frac{1}{\sqrt{2a}}.$$

44. (b) $\lim_{x\to 2} \frac{x^n-2^n}{x-2} = n.2^{n-1} \Rightarrow n.2^{n-1} = 80 \Rightarrow n = 5$.

45. (a) $\lim_{x\to 0} \frac{x.2\sin^2 x}{x^2} = 2.\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2 \cdot \lim_{x\to 0} x = 0$.

46. (c) $\left[\lim_{x\to\infty}\left(1+\frac{1}{x/2}\right)^{x/2}\right]^2 = e^2$.

47. (a) $\lim_{x\to 1} \frac{(2x-3)(\sqrt{x}-1)\times(\sqrt{x}+1)}{(x-1)(2x+3)\times(\sqrt{x}+1)} = \frac{-1}{5.2} = \frac{-1}{10}$.

48. (d) $\lim_{x\to 0} \frac{2\sin 2x}{2x} = 2$.

49. (a) $\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} = \lim_{h\to 0} \frac{(\sqrt{x+h})^2-(\sqrt{x})^2}{h(\sqrt{x+h}+\sqrt{x})} = \frac{1}{2\sqrt{x}}$.

Aliter : Apply L-Hospital rule,

$$\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} = \lim_{h\to 0} \frac{1}{2\sqrt{x+h}} = \frac{1}{2\sqrt{x}}.$$

50. (b) $\lim_{x\to 0} \frac{2^x-1}{(1+x)^{1/2}-1} = \lim_{x\to 0} \frac{2^x \log 2}{\frac{1}{2}(1+x)^{-1/2}}$

$$\left\{\because \lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}\right\}$$

$= 2\log 2 = \log 4$.

51. (c) $\lim_{x\to 0} \frac{1-\cos mx}{1-\cos nx} = \lim_{x\to 0} \left\{\frac{2\sin^2 \frac{mx}{2}}{2\sin^2 \frac{nx}{2}}\right\}$

$$= \lim_{x\to 0} \left[\left\{\frac{\sin\frac{mx}{2}}{\frac{mx}{2}}\right\}^2 \frac{m^2x^2}{4} . \frac{1}{\left\{\frac{\sin\frac{nx}{2}}{\frac{nx}{2}}\right\}^2} . \frac{4}{n^2x^2}\right]$$

$$= \frac{m^2}{n^2} \times 1 = \frac{m^2}{n^2}.$$

Aliter : Apply L-Hospital's rule,

$$\lim_{x\to 0} \frac{1-\cos mx}{1-\cos nx} = \lim_{x\to 0} \frac{m\sin mx}{n\sin nx} = \lim_{x\to 0} \frac{m^2\cos mx}{n^2\cos nx} = \frac{m^2}{n^2}.$$

52. (c) $\lim_{x\to 1} \frac{x-1}{(x-1)(2x-5)} = -\frac{1}{3}$.

Aliter : Apply L-Hospital's rule.

53. (b) Apply L-Hospital's rule, $\lim_{x\to 0} \frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$

$$= \lim_{x\to 0} \frac{\cos x}{2\sqrt{1+\sin x}} + \frac{\cos x}{2\sqrt{1-\sin x}} = \frac{1}{2} + \frac{1}{2} = 1.$$

54. (a) $\lim_{\alpha\to\pi/4} \frac{\sin\alpha-\cos\alpha}{\alpha-\pi/4}$

$$= \lim_{\alpha\to\pi/4} \left\{\frac{\sqrt{2}\left(\sin\alpha.\frac{1}{\sqrt{2}}-\cos\alpha.\frac{1}{\sqrt{2}}\right)}{\left(\alpha-\frac{\pi}{4}\right)}\right\}$$

$$= \sqrt{2} \lim_{\alpha \to \pi/4} \frac{\sin\left(\alpha - \frac{\pi}{4}\right)}{\left(\alpha - \frac{\pi}{4}\right)} = \sqrt{2} \times 1 = \sqrt{2}.$$

Aliter : Apply L-Hospital's rule,

$$\lim_{\alpha \to \pi/4} \frac{\sin\alpha - \cos\alpha}{\alpha - (\pi/4)} = \lim_{\alpha \to \pi/4} \frac{\cos\alpha + \sin\alpha}{1} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}.$$

55. (c) $\lim_{x \to 0} \frac{\tan 2x - x}{3x - \sin x} = \lim_{x \to 0} \left\{ \frac{\frac{2\tan 2x}{2x} - 1}{3 - \frac{\sin x}{x}} \right\} = \frac{1}{2}.$

Aliter : Apply L-Hospital's rule

$$\lim_{x \to 0} \frac{\tan 2x - x}{3x - \sin x} = \lim_{x \to 0} \frac{2\sec^2 2x - 1}{3 - \cos x} = \frac{2-1}{3-1} = \frac{1}{2}.$$

56. (a) $\lim_{x \to 0} \frac{\sin ax}{\sin bx} = \lim_{x \to 0} \frac{a}{b} \frac{\sin ax}{ax} \frac{bx}{\sin bx} = \frac{a}{b}.$

57. (b) $\lim_{x \to 0} \frac{\sin x^o}{x} = \lim_{x \to 0} \frac{\sin \frac{\pi x}{180}}{x} = \frac{\pi}{180}$

$\left\{ \because x^\circ = \frac{\pi x}{180} \textit{radian} \right\}.$

58. (a) Apply the L-Hospital's rule, $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}.$

59. (b) $\lim_{x \to 0} \frac{\cos ax - \cos bx}{x^2}$

$$= \lim_{x \to 0} \frac{2\sin\left(\frac{a+b}{2}\right)x . \sin\left(\frac{b-a}{2}\right)x}{\left(\frac{a+b}{2}\right)x . \frac{2}{a+b} . \frac{2}{b-a} . \left(\frac{b-a}{2}\right)x} = \frac{b^2 - a^2}{2}$$

Aliter : Apply L-Hospital's rule,

$$\lim_{x \to 0} \frac{\cos ax - \cos bx}{x^2} = \lim_{x \to 0} \frac{-a\sin ax + b\sin bx}{2x}$$

$$= \lim_{x \to 0} \frac{-a^2\cos ax + b^2\cos bx}{2} = \frac{b^2 - a^2}{2}.$$

60. (a) $\lim_{x \to a} \frac{x^9 + a^9}{x + a} = 9 \Rightarrow \frac{2a^9}{2a} = 9 \Rightarrow a^8 = 9$

$\Rightarrow a = 9^{1/8}$